ESCI 6000 / INES 8090 – NUMERICAL WEATHER PREDICTION – HOMEWORK #1

DUE: FRIDAY, SEPTEMER 12, 2008

CHAPTER 6:

Derive the system of equations for a dry, hydrostatic atmosphere, for $s=\sigma=p/p_s$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \dot{s}\frac{\partial u}{\partial s} - \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\partial \phi}{\partial x} + fv$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \dot{s}\frac{\partial v}{\partial s} - \frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\partial \phi}{\partial y} - fu$$

$$\frac{\partial p}{\partial s}\frac{\partial s}{\partial z} = -\rho g$$

$$\frac{d}{dt}\left(\ln(\rho\frac{\partial z}{\partial s})\right) + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \dot{s}}{\partial s} = 0$$

CHAPTER 10:

(a) Derive a five-point, fourth-order accurate, centered-in-space, finite-difference approximation for $\delta f/\delta x$ using Taylor series. Show that:

$$\frac{\partial f}{\partial x} = \frac{1}{2\Delta x}\left[\frac{4}{3}\left(f(x+\Delta x) - f(x-\Delta x)\right) - \frac{1}{6}\left(f(x+2\Delta x) - f(x-2\Delta x)\right)\right]$$

(b) Use $f(x)=A\cos(kx)$, where $k=2\pi/L$ and the equation above to show that the response function for the 5-point, fourth-order, centered-in-space finite difference approximation for $\delta f/\delta x$ is

$$R = \frac{\sin(k\Delta x)}{k\Delta x}\left(\frac{4}{3} - \frac{1}{3}\cos(k\Delta x)\right)$$

Recall that for the 3-point, second-order, centered-in-space approximation for $\delta f/\delta x$,

$$R = \frac{\sin(k\Delta x)}{k\Delta x}$$

(c) Plot R for both the fourth-order scheme and the second-order scheme, as a function of $\Delta x/L$, and interpret the results.